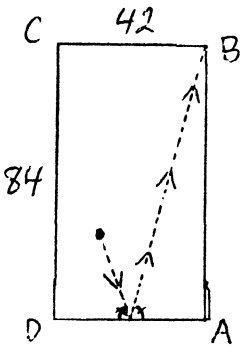
ROUND I: Similarity and Pythagorean relationships

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. I have two similar right triangles. The first has legs of 12 and 5. The second has a hypotenuse of 156. What is the length of its shorter leg?

2. A rectangular pool table is 42 inches long and 84 inches wide. The ball is located 12 in. from $\overline{DC}$ and 21 in. from $\overline{DA}$. You wish to hit the ball as shown in the drawing. How far from point A should the ball hit $\overline{DA}$?



3. The lengths of the sides of a triangle are in the ratio 3:4:5. If the length of one of the altitudes of this triangle is 60, what is the greatest possible area of the triangle?

ANSWERS

1. (1 pt) _____

2. (2 pts) _____ inches

3. (3 pts) _____

Bartlett, Leicester, Mass. Academy

February 2, 2000 WOCOMAL VARSITY MEET

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve: $2(2x+2)+2=2^2$.

2. The area of a rectangle is $4x^2-196x$. The length and width are increased by 9 each. What is the area of the larger rectangle? Give your answer as a polynomial in x.

and the width is 2x-14.

3. A merchant visited three fairs. At the first he doubled his money and then spent \$30. At the second he tripled his remaining money and then spent \$54. At the third he quadrupled the remaining money and then spent \$72. He had \$48 left. How much money did he start with?

ANSWERS

1. (1 pt) $x =$ _____

2. (2 pts) _____

3. (3 pts) \$ _____

Bromfield, Shepherd Hill, South

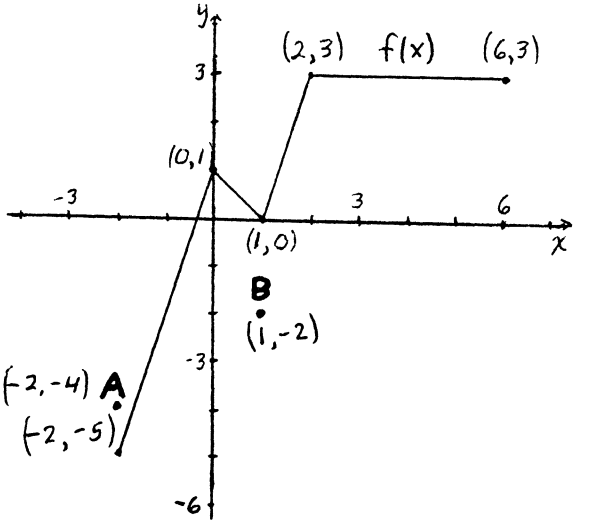
ROUND III: Functions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The equation $g(x) = 0$ has roots $x = -1$ and $x = 4$. List all the roots of the equation $xg(x) = 0$.

2. If $f(x) = 2x^2 + 5$ and $g(x) = 3x + c$, find all values of c so that the graph of $f(g(x))$ crosses the y-axis at (0,23).

3. Given f(x) by the graph at the right, let $g(x) = f(x+a) + b$. Find values for a and b that make the graph of g(x) pass through points A and B.



ANSWERS

1. (1 pt) $x =$ _____

2. (2 pts) $c =$ _____

3. (3 pts) $a =$ _____ $b =$ _____

Hudson, Mass. Academy

ROUND IV: Combinatorics

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. In how many ways can 7 people be seated in a room containing 10 chairs if who sits where does matter?

2. Find n and r if ${}_nP_r = 60$ and ${}_nC_r = 10$.

3. Four people sit on a park bench. Two of them, Ann Marie and Justin, are in love and insist on sitting next to each other. Determine the number of possible seating arrangements with the above condition.

ANSWERS

1. (1 pt) _____

2. (2 pts) n = _____ r = _____

3. (3 pts) _____

Algonquin, Clinton, Hudson

ROUND V: Analytic geometry of straight lines and conic sections

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. The elliptical orbit of the planet Jupiter has the center of the sun as one focus. If the closest distance to the center of the sun is 460 million miles and the farthest distance is 508 million miles, how far from the center of the sun is the second focus?

2. Find the exact distance between the lines $x+2y=6$ and $y=-\frac{1}{2}x+\frac{11}{2}$.

3. Consider the point(s) of intersection of the parabola $y=x^2-6x+9$ and the line $y=x+3$. What are the coordinates of the center of the circle that passes through the point(s) considered above if the center's x-coordinate is 50% larger than its y-coordinate?

ANSWERS

1. (1 pt) million miles

2. (2 pts) _____

3. (3 pts) (,)

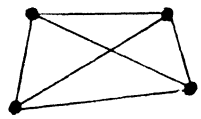
Assabet Valley, Leicester, Mass. Academy

TEAM ROUND: Related problem solving

ALL ANSWERS MUST BE DIAGRAMS OR WRITTEN AS POSITIVE INTEGERS AND ON THE SEPARATE TEAM ROUND ANSWER SHEET 2 points each

Several points with no three collinear are arranged in a plane. Each point is joined to every other point by a line segment. The crossing number for a particular arrangement is the number of distinct points of intersection of these segments, not including their endpoints.

For example, here is an arrangement of 4 points with a crossing number of 1.



For 3 points the crossing number must be 0.

1. How many line segments are determined by 4 points as described above?

2. Make a drawing showing 4 points and all the line segments so that the crossing number is 0. Make the 4 points very clear and draw against something like a calculator edge to get straight segments.

3. How many segments are determined by 5 points as above?

4. Make a drawing showing 5 points and all the line segments so that the crossing number is 3. Make the 5 points very clear and join them with straight segments.

5. What is the minimum crossing number for 5 points?

6. What is the maximun crossing number for 5 points?

7. How many segments are determined by 6 points as above?

8. What is the minimum crossing number for 6 points?

9. What is the maximum crossing number for 6 points?

February 2, 2000 WOCOMAL Varsity Meet ANSWERS

ROUND I: (1 pt) 1. 60

sim Pyth (2 pts) 2. 24 in

(3 pts) 3. 3750

ROUND II (1 pt) 1. $-\frac{1}{2}$ or $-.5$

alg 1 (2 pts) 2. $4x^2+36x-115$

(3 pts) 3. $29

ROUND III (1 pt) 1. $-1, 0, 4$ any order

Funct (2 pts) 2. $-3, 3$ or ± 3

(3 pts) 3. Either $a = 2$, $b = -5$ OR $a = 3\frac{1}{3}$, $b = -5$

ROUND IV (1 pt) 1. 604,800

comb (2 pts) 2. $n = 5$, $r = 3$ need both

(3 pts) 3. 12

ROUND V (1 pt) 1. 48 million miles

analyt (2 pts) 2. $\sqrt{5}$ No decimals

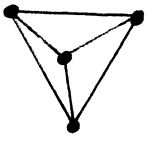
(3 pts) 3. $(6, 4)$

TEAM ROUND 2 pts each

1. 6

Need triangle with 1 interior pt and 6 segments not intersecting

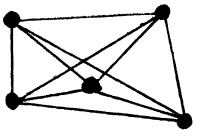
Eg. 2.



3. 10

Need quadrilateral with 1 interior pt and 10 segments intersecting at 3 pts

Eg. 4.



5. 1

6. 5

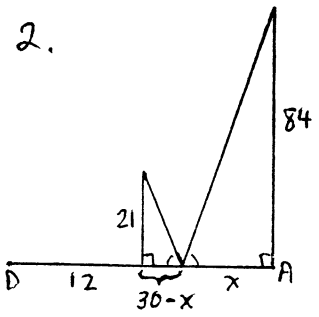
7. 15

8. 3

9. 15

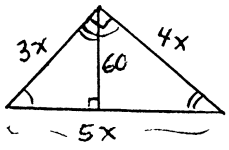
ROUND I

1. $5^2+12^2=c^2 \Rightarrow c=13$ = 1st hypot.
Then $\frac{5}{x}=\frac{13}{156} \Rightarrow x=60$

2. 

Make $\sim\Delta$'s
$\frac{x}{30-x}=\frac{84}{21}$
$x=4(30-x)$
$x=24$ in

3. 3:4:5 makes right Δ
Since alt = 60 is given, max area comes from using longest side, hypotenuse, as base.
We get $\sim$ 3,4,5 rt Δ's



$\frac{4x}{60}=\frac{5x}{3x}$
$\frac{x}{15}=\frac{5}{3}$
$x=25$ and $5x=125$.
Δ area $=\frac{1}{2}\cdot 125\cdot 60 = 3750$

ROUND II

1. $\div 2$ gets $2x+2+1=2 \Rightarrow x=-\frac{1}{2}$

2. $4x^2-196=(2x+14)(2x-14)$
length width
Larger: $(2x+23)(2x-5)$
$=4x^2+36x-115$

3. Let x = what he started with
After 1st: $2x-30$
" 2nd: $3(2x-30)-54 = 6x-144$
" 3rd $4(6x-144)-72 = 48$
$6x-144=30$
$x=29$

ROUND III

1. $x=0$ works, as do the given roots of $g(x)=0$.

2. $f(g(x))=2(3x+c)^2+5$
$=18x^2+12cx+2c^2+5$
$f(g(0))=2c^2+5=23$
$c^2=9 \Rightarrow c=\pm 3$

3. g graph $\cong$ f graph, but shifted left or right by a and up or down by b. Slope of $\overline{AB}=\frac{2}{3}$. We need 2 pts on f graph which when connected give slope $\frac{2}{3}$ and which are the proper distance apart, AB
There are 2 such pairs of pts, $(0,1),(3,3)$ and $(1\frac{1}{3},1),(4\frac{1}{3},3)$.
Thus shift left 2 $(a=2)$ and down 5 $(b=-5)$ or shift left $3\frac{1}{3}$ $(a=3\frac{1}{3})$ and down 5 $(b=-5)$.

ROUND IV

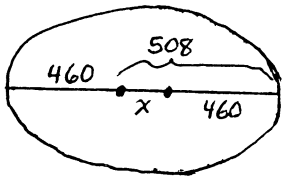
1. $10P_7 = 604{,}800$

2. In general ${}_nP_r = r!\cdot {}_nC_r$.
$\therefore 60 = r!\cdot 10 \Rightarrow r!=6$ and $r=3$
Try $n=4,5,\ldots$ $n=5$ works

3. First to keep AM and J together, think of permutations of just 3 entities. There are $3!=6$ of these. But for each, AM and J can switch places, so there are twice as many, 12.
Alt: list all cases

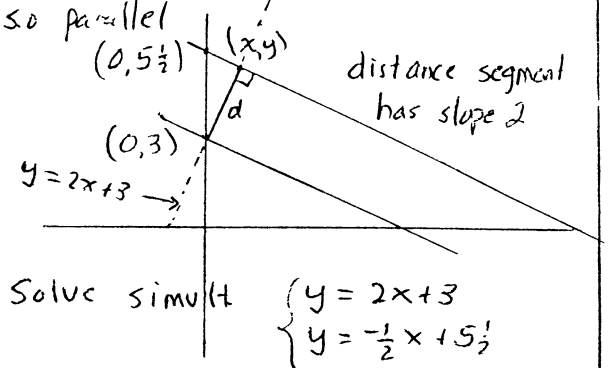
ROUND V

1.



$x + 460 = 508 \Rightarrow x = 48$

2. $y = -\frac{1}{2}x + 3$ and $y = -\frac{1}{2}x + 5\frac{1}{2}$



Solve simult $\begin{cases} y = 2x + 3 \\ y = -\frac{1}{2}x + 5\frac{1}{2} \end{cases}$

$2x + 3 = -\frac{1}{2}x + 5\frac{1}{2} \Rightarrow x = 1, y = 5$

Dist $d = \sqrt{(1-0)^2 + (5-3)^2} = \sqrt{5}$

OR use a formula for dist. between a pt and a line

3. $x^2 - 6x + 9 = x + 3$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$x = 6$ or 1

Intersection pts: $A(6,9)$ and $B(1,4)$.

Let circle center be $C(1.5k, k)$

$CA^2 = CB^2$

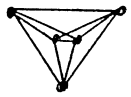
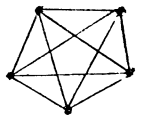
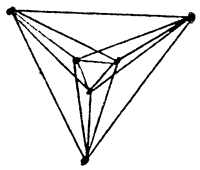
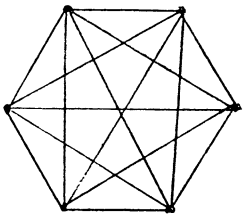
$(1.5k-6)^2 + (k-9)^2 = (1.5k-1)^2 + (k-4)^2$

$-18k + 36 - 18k + 81 = -3k + 1 - 8k + 16$

$100 = 25k \Rightarrow k = 4$

Center $(6, 4)$

TEAM ROUND

1. ${}_4C_2 = 6$ or just count them
2. See ans
3. ${}_5C_2 = 10$ or count them
4. see ans
5. 1. 
6. 5 
7. ${}_6C_2 = 15$ or count them
8. 3 
9. 15 

Use a non-regular hexagon to get 3 near the middle and 12 others